

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT3

Branch: B. Tech (All)

Semester : 1

Date : 26/02/2020

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a)** If $y = \sin 2x \cos 2x$ then y_n equal to
 (A) $\frac{1}{2}(4)^n \cos\left(\frac{n\pi}{2} + 4x\right)$ (B) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 4x\right)$
 (C) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 2x\right)$ (D) none of these
- b)** If $y = \frac{3x+2}{2x+3}$, then y_n equal to
 (A) $\frac{5(-1)^{n+1} n! 2^{n-1}}{(2x+3)^{n+1}}$ (B) $\frac{5(-1)^n n! 2^n}{(2x+3)^{n+1}}$ (C) $\frac{5(-1)^n n! 2^n}{(2x+3)^{n-1}}$
 (D) none of these
- c)** If $y = \log(1+x)$, then x equal to
 (A) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!}$ (B) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$
 (C) $y + \frac{y^2}{2!} + \frac{y^3}{3!} + \frac{y^4}{4!} + \dots$ (D) none of these
- d)** $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \infty$ represent expansion of
 (A) $\sinh x$ (B) $\cosh x$ (C) $\cos x$ (D) e^x
- e)** $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \underline{\hspace{2cm}}$
 (A) $a - b$ (B) $\log(a - b)$ (C) $\log ab$ (D) $\log\left(\frac{a}{b}\right)$
- f)** $\lim_{x \rightarrow \infty} \frac{x^n}{e^{ax}}$ (n being a positive integer and $a > 0$) = $\underline{\hspace{2cm}}$



- (A) -1 (B) 0 (C) 1 (D) None of these
- g) If $P = r \tan \theta$, then $\frac{\partial P}{\partial r}$ is equal to
 (A) $\sec^2 \theta$ (B) $\tan \theta$ (C) $\tan \theta + r \sec^2 \theta$ (D) $\frac{1}{2} \tan \theta$
- h) If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to
 (A) $\frac{\partial f / \partial x}{\partial f / \partial y}$ (B) $\frac{\partial f / \partial y}{\partial f / \partial x}$ (C) $-\frac{\partial f / \partial y}{\partial f / \partial x}$ (D) $-\frac{\partial f / \partial x}{\partial f / \partial y}$
- i) If $u(x, y, z) = 0$ then the value of $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$ is equal to
 (A) 1 (B) -1 (C) 0 (D) none of these
- j) If $f_1 = \frac{vw}{u}$, $f_2 = \frac{wu}{v}$, $f_3 = \frac{uv}{w}$; then $\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)}$ is equal to
 (A) 0 (B) 1 (C) 3 (D) none of these
- k) The value of $\sum_{k=1}^{10} \left(\sin \frac{2\pi k}{11} - i \cos \frac{2\pi k}{11} \right)$ is
 (A) -1 (B) 0 (C) -i (D) i
- l) If $\left(\frac{1-i}{1+i} \right)^{100} = a + ib$, then
 (A) $a = 2, b = -1$ (B) $a = 1, b = 0$ (C) $a = 0, b = 1$ (D) $a = -1, b = 2$
- m) If A is a non-zero column vector $(n \times 1)$, then the rank of matrix AA^T is
 (A) 0 (B) 1 (C) $n-1$ (D) n
- n) An eigenvalue of a square matrix A is $\lambda = 0$. Then
 (A) $|A| \neq 0$ (B) A is symmetric (C) A is singular
 (D) A is skew-symmetric

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $y = \tan^{-1} x$ then find the value of $y_n(0)$. (5)

b) Expand $f(x) = \frac{e^x}{e^x + 1}$ in powers of x up to x^3 by Maclaurin's series. (5)

c) If $V = \frac{1}{r}$ where $r^2 = x^2 + y^2 + z^2$ then show that $V(x, y, z)$ satisfies (4)

$$\text{Laplace's equation } \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

Q-3 Attempt all questions (14)

a) If $y = a \cos(\log x) + b \sin(\log x)$ then prove that (5)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

b) Prove that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$ (5)



c) Evaluate: $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$. (4)

Q-4 Attempt all questions (14)

a) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ (5)

b) If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. (5)

c) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in powers of $(x-3)$. (4)

Q-5 Attempt all questions (14)

a) If $u = \tan^{-1} \left(\frac{x^2 + y^2}{x - y} \right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$. (5)

b) Evaluate: $\lim_{x \rightarrow 0} \frac{a}{x^2} \left[\frac{\sin kx}{\sin lx} - \frac{k}{l} \right]$ (5)

c) If $y = \frac{x^4}{(x-1)(x-2)}$ then find y_n . (4)

Q-6 Attempt all questions (14)

a) Using the formula $R = \frac{E}{I}$, find the maximum error and percentage of error in R if $I = 20$ with a possible error of 0.1 and $E = 120$ with a possible error of 0.05 and $R = 6$. (5)

b) Expand $\sin^5 \theta \cos^2 \theta$ in a series of sines of multiples of θ . (5)

c) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by Gauss-Jordan reduction method. (4)

Q-7 Attempt all questions (14)

a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. (5)

b) Find the continued product of all the values of $\left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^{\frac{3}{4}}$. (5)

c) If $\tan(\alpha + i\beta) = x + iy$ then prove that $x^2 + y^2 + 2x \cot 2\alpha = 1$. (4)

Q-8 Attempt all questions (14)

a) Investigate for what values of λ and μ the equations (5)

$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, have (i) no solution
(ii) a unique solution (iii) an infinite number of solutions.

b) Find the fourth roots of unity and sketch them on the unit circle. (5)

c) Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. (4)

